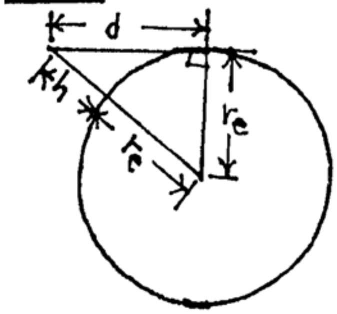
Chapter 1

1-2.



$(h+r_e)^2 = d^2 + r_e^2$

$\Rightarrow h^2 + 2hr_e = d^2$ where $h \ll r_e$

$\Rightarrow d^2 \approx 2hr_e$ and $r_e = \frac{4}{3}(3960 \text{ miles}) = 5280 \text{ miles}$

Let h = antenna height in feet, and d in miles.

$\Rightarrow d^2 (\text{miles})^2 = 2h \cancel{(\text{feet})} \left(\frac{1 \text{ mile}}{5280 \cancel{\text{feet}}}\right)(5280 \text{ miles})$

$\Rightarrow d^2 = 2h$ or $\underline{\underline{d = \sqrt{2h}}}$

1-5. $\{m_j\} = \{-1.0, 0.0, 3.0, 4.0\}$ $i = 1, 4$; $P_1 = P_2 = 0.2$; $P_3 = P_4 = 0.3$

$\Rightarrow I_1 = I_2 = \frac{-\ln(0.2)}{\ln 2} = 2.322$ bits, $I_3 = I_4 = \frac{-\ln(0.3)}{\ln 2} = 1.737$ bits

$H = \sum_{j=1}^{m} P_j I_j = 2\left[(0.2)(2.322) + (0.3)(1.737)\right] = \underline{\underline{1.971 \text{ bits}}}$

1-8. Let p_1 = prob. of sending a binary 1

(a) p_2 = prob. of sending a binary 0 $= 1 - p_1$

$H = \sum_{i=1}^{2} P_i I_i = p_1 \log_2\left(\frac{1}{p_1}\right) + (1-p_1)\log_2\left(\frac{1}{1-p_1}\right)$

$H = \frac{1}{\ln 2}\left[-p_1 \ln(p_1) - (1-p_1)\ln(1-p_1)\right]$

$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - \left((-1)\ln(1-p_1) + \frac{1-p_1}{1-p_1}(-1)\right) = 0$

$\Rightarrow -\ln p_1 - \cancel{1} + \ln(1-p_1) + \cancel{1} = 0$

or $\ln\left(\frac{1-p_1}{p_1}\right) = 0 = \ln 1$

thus $\frac{1}{p_1} - 1 = 1 \Rightarrow \underline{\underline{p_1 = \frac{1}{2} = p_2}}$

(b) $H_{max} = \frac{1}{2}\log_2 2 + \left(1 - \frac{1}{2}\right)\log_2 2 = \underline{\underline{1 \text{ bit}}}$

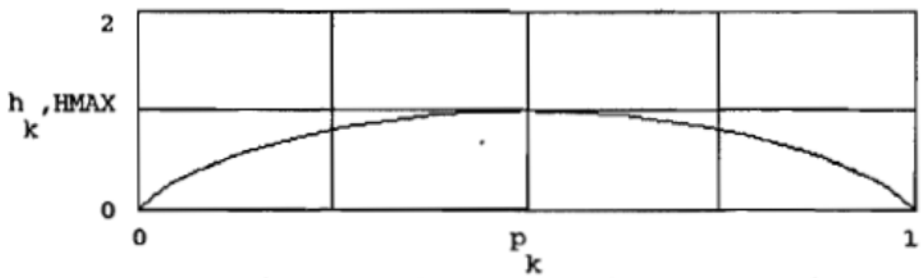
Math CAD Solution

LET p=The probability for sending a binary 1, then the probability for sending a binary 0 is (1-p). From the entropy formula for H(p), we can draw the figure of H(p) ,and from this figure , we can find the maximum entropy and the p.

H(p)=(p*ln(P)+(1-p)*ln(1-p))/(-ln(2))

$k \equiv 0\,..50 \qquad p_k := \dfrac{k}{50} \qquad HMAX := 1$

$$h_k := \frac{-1}{\ln(2)}\left[p_k\ \ln\left[p_k\right] + \left[1-p_k\right]\cdot\ln\left[1-p_k\right]\right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-9. $P_1 = 0.25\ ;\ P_2 = P_3 = 0.15\ ;$

$P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = 0.07$

$0.25 + 2(0.15) + 6(0.07) = 0.97$

$\therefore\ P_{10} = 0.03 \quad \text{since} \sum_{j=1}^{10} P_j = 1.0$

$$H = \sum_{j=1}^{M} P_j \log_2\left(\frac{1}{P_j}\right) = \left[\frac{-1}{\ln 2}\right]\sum_{j=1}^{10} P_j \ln P_j$$

$$= \left[\frac{-1}{\ln 2}\right]\left[.25\ln .25 + (2).15\ln .15 + (6).07\ln .07 + .03\ln .03\right]$$

$\underline{\underline{H = 3.084 \text{ bits}}}$

1-11.

$M = 10 \qquad P_j = \frac{1}{10} \quad j = 1, 10 \qquad R = \frac{H}{T} = 2\frac{b}{s}$

$$H = \frac{-10(.1)\ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{2 \text{ bits/sec}} = \underline{\underline{1.661 \text{ sec.} = T}}$$

1-14.

(a) chars := 110 Number of characters available

$b := \text{ceil}\left[\frac{\log(\text{chars})}{\log(2)}\right]$ Number of bits required to represent a character

$\Longrightarrow$ b = 7 bits

(b) B := 3200 Hz Channel bandwidth
SNRdB := 20 dB Signal to noise ratio

$SNR := 10^{\frac{SNRdB}{10}}$ $\Longrightarrow$ SNR = 100 (Absolute power ratio)

$C := B\cdot\left[\frac{\log(1 + SNR)}{\log(2)}\right]$ $\Longrightarrow$ $C = 2.131\cdot 10^4$ Channel capacity (bits/sec)

$c := \frac{C}{b}$ $\Longrightarrow$ $c = 3.044\ 10^3$ Channel capacity (chars/sec)

(c) Assuming equally likely characters,
information content of each character is:

$P := \frac{1}{\text{chars}}$ Probability of each character

$I := \frac{\log\left[\frac{1}{P}\right]}{\log(2)}$ $\Longrightarrow$ I = 6.781 bits

1-17.

$x_0 := 1 \quad x_1 := 0 \quad x_2 := 1 \quad x_3 := 1 \quad x_4 := 1$ Input vector

$ga_0 := 1 \quad ga_1 := 0 \quad ga_2 := 0 \quad ga_3 := 1$ Gain vector, mod2 adder

$gb_0 := 1 \quad gb_1 := 1 \quad gb_2 := 1 \quad gb_3 := 1$ Gain vector, mod2 adder

$k := 0 \,..\, \text{length}(ga) - 2$

$v_k := 0 \qquad k := 0 \,..\, \text{length}(x) - 1 \qquad v_{k+\text{length}(ga)-1} := x_k$

$k := \text{length}(x) + \text{length}(ga) - 1 \,..\, \text{length}(x) + 2\,\text{length}(ga) - 3$

$v_k := 0 \qquad i := 0 \,..\, \text{length}(v) - \text{length}(ga)$

$j := 0 \,..\, \text{length}(ga) - 1$

$$sa_i := \sum_j \left[ga_{\text{length}(ga)-j-1}\, v_{j+i}\right]$$

$$sa_i := \text{mod}\left[sa_i, 2\right]$$

$$sb_i := \sum_j \left[gb_{\text{length}(gb)-j-1}\, v_{j+i}\right]$$

$$sb_i := \text{mod}\left[sb_i, 2\right] \qquad s_{2\,i} := sa_i \qquad s_{2\,i+1} := sb_i$$

$i := 0 \,..\, 2\,\text{length}(x) - 1$

$out_i := s_i$

For $x^T = (1 \;\; 0 \;\; 1 \;\; 1 \;\; 1)$

====> $out^T = (1 \;\; 1 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 1)$